PROPAGATION OF OSCILLATIONS IN TWO-PHASE DISPERSE STRUCTURED SYSTEMS*

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A mathematical model of the process of propagation of oscillations in two-phase disperse systems is proposed, taking into account air liberation, and an algorithm of numerical integration is obtained.

In [1-6] results of investigations connected with the behavior of two-phase disperse structured systems in the case of vibratory actions are presented.

The aim of the present investigation is the development of a mathematical model of the process of propagations of oscillations in two-phase structured disperse systems, where one of the phases is liquid and the other gaseous, and the investigation of the laws of this process.

We shall model a two-phase system in the form of a viscoelastic medium in which there propagate vertically directed oscillations according to the law $e = -A \cos\omega t$, where e is the displacement, A is the amplitude of oscillations, ω is the angular velocity, and t is time.

We denote the modulus of elasticity of the two-phase system by E(x, t).

The variability of the modulus of elasticity confirms that the process of propagation of oscillations takes palce with a continuous variation of the density of the oscillating medium, this being a function of the coordinates and time, and leading to a variation of the velocity of sound, i.e., the front of the disturbance wave in the process of vibration of the system.

The variable density of the medium and the velocity of sound in the given case are functions of the nonuniform distribution of air concentration in the vibrating structured system. The nonuniform distribution in the medium is explained by liberation of the trapped air in the vibration process; at the same time, the pattern of distribution of air bubbles in the structured system varies with time.

When developing the mathematical model of propagation of oscillations in two-phase systems, we have made the following assumption: the air bubbles are taken as sphere-shaped and with the same dimensions which do not change in the process of liberation of bubbles, i.e., the density of air inside the bubble is constant.

In addition, the velocity of motion of an air bubble relative to the system being vibrated has the form [7]

$$v_{\text{b}} = \frac{d^2 g}{18\nu}. \quad (1)$$

We shall assume that before the start of oscillations the distribution of air bubbles in the two-phase system at all points will be uniform. In the given case the one-dimensional mathematical model of propagation of oscillations has the form [8]

$$\frac{\partial v}{\partial t} = c^2 \frac{\partial^2 e}{\partial x^2} + \nu \frac{\partial^2 v}{\partial x^2}. \quad (2)$$

The variation of the air bubble concentration during oscillations of the system is determined from the diffusion equation (of Fokker–Planck) [9]

$$\frac{\partial w}{\partial t} = -v_{\text{b}} \frac{\partial w}{\partial x} + D \frac{\partial^2 w}{\partial x^2}. \quad (3)$$

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The term $w(\partial v_b/\partial x)$ in Eq. (3) is neglected because of its smallness.

Thus, the original mathematical model describing propagation of oscillations in the two-phase medium has the form

$$\frac{\partial v}{\partial t} = c^2 \frac{\partial^2 e}{\partial x^2} + \nu \frac{\partial^2 v}{\partial x^2},$$
$$v_b = \frac{d^2 g}{18\nu}, \tag{4}$$
$$\frac{\partial w}{\partial t} = -v_b \frac{\partial w}{\partial x} + D \frac{\partial^2 w}{\partial x^2},$$

where

$$c^2 = E(x,\ t)/\rho = \text{var};\quad \rho = f_1(w);\quad c = f_2(w);\quad \nu = \mu/\rho = \text{var};\quad \nu = f_3(A,\ \omega,\ w).$$

The process of numerical integration of system (4) consists of the following. Specifying the matrix of initial volume concentrations of air bubbles at all nodes of integration according to the second equation of system (4), we determine the velocity of the bubble for the given node at the given instant of time. We then substitute the velocity thus obtained into the third equation of system (4) and determine the new value of air concentration at the given node after a step of integration with respect to time.

With the new value of concentration taken into account, we find the velocity of sound, the modulus of elasticity, and the coefficient of kinematic viscosity, after which according to the first equation of system (4) we determine the rate of displacement of an elemental volume of the medium for a new instant of time. All these three equations of system (4) we solve simultaneously step after step at all nodes over the height of the column and over time, as long as we have not obtained a steady-state process of solution of the vibration equation. In the solution process the air concentration at the nodes of the grid of integration tends to zero.

The density of the two-phase medium was determined from the expression $\rho = \rho_{l.ph}(1 - w)$, and the modulus of elasticity from

$$E(x,\ t) = E_{l.ph}/[1 + (E_{l.ph}/E_a)\, w].$$

From the values of density and modulus of elasticity thus computed, we determine the velocity of sound in the medium:

$$c^2 = \frac{c_{l.ph}^2}{(1-w)\left[1 + \left(\frac{c_{l.ph}}{c_a}\right)^2 \frac{\rho_{l.ph}}{\rho_a} w\right]}. \tag{5}$$

In Eq. (5) instead of E_a we have introduced $c_a^2\rho_a$, while instead of $E_{l.ph}$ we take $c_{l.ph}^2\rho_{l.ph}$. The stability condition of the solution of the first equation of system (4) is provided by $\delta x \leq c\delta t$ [9].

Physically such a condition is not true, since for one integration step in time the disturbance wave propagates one integration step along the length, i.e., fulfillment of the equality $\delta x = c\delta t$ is necessary. The second term on the right side of this equation is a viscosity constituent.

Viscous friction (internal friction) is friction between layers of the oscillating solid medium; therefore, Eq. (2) does not correspond to the physics of the phenomenon under consideration. It is proposed by us to take $\nu(d^2v/dy^2)$ instead of the second term on the right side of Eq. (2). This means that we consider oscillations in the central symmetrical layer of a plane problem.

Then, taking into account the fact that $\delta x = \delta y = c\delta t$, we represent this equation in the finite-difference form

$$\Delta_t^2 \bar{e} = \Delta_x^2 \bar{e} + \frac{\nu}{\delta t c^2}\, \Delta_t \Delta_y^2 \bar{e}, \tag{6}$$

where $\Delta_t^2\bar{e} = E_0 - 2E2_0 + E1_0$; $\Delta_x^2\bar{e} = E2_+ - 2E2_0 + E2_-$; E1, E2, E are, respectively, the values of the dimensionless displacements per step in time back and per step in time forward.

The indices +, 0, — denote the values of the dimensionless displacement per step in length forward, at the given node, and per step in length back. For the numerical integration of Eq. (6) we introduce two fictitious layers parallel to and symmetric about the central layer.

Taking into account the fact that we consider the central layer of a plane problem, we assume that the viscous friction in the fictitious layers must be the same, i.e.,

$$E2'_{+} = E2'_{-} = E2_0(1-\alpha), \quad E1'_{+} = E1'_{-} = E1_0(1-\alpha), \tag{7}$$

where α is a coefficient characterizing the measure of friction between the layers; $E2'_{+}$, $E2'_{-}$, $E1'_{+}$, $E1'_{-}$ are, respectively, the dimensionless displacements in the fictitious layers per step in length forward and back along the y axis for the given instant of time, and per step in time back. With the results of [5] taken into account, α = 0.05-0.15. Then Eq. (6) assumes the form

$$E_0 = E2_{+} + E2_{-} - E1_0 + 2\alpha D_1(E1_0 - E2_0), \tag{8}$$

where $D_1 = \nu\omega K_t/2\pi c^2$.

From the dimensionless displacements we determined the dimensionless velocities according to the formula

$$u = \frac{E_{+} - E_{-}}{2\bar{\delta t}}, \text{ where } \bar{\delta t} = \delta t\omega.$$

The finite-difference analog of the third equation of system (4) has the form

$$w_1 = w_0(1-2D_0) + w_2\left(D_0 - \frac{v_b}{c}\right) + w_4\left(D_0 + \frac{v_b}{c}\right), \tag{9}$$

where $D_0 = D\omega K_t/2\pi c^2$.

The indices 0, 2, and 4 of w denote the concentration of air along the height of the column, while the index 1 denotes the concentration of air at the given node for the new instant of time.

We denote $p_0 = 1 - 2D_0$; $p = D_0 - v_b/c$; $q = D_0 + v_b/c$, where, proceeding from the theory of Markov processes [10], p_0 is the measure of transfer of concentration from the zeroth node into the same node for the new instant of time; p is the measure of transfer of the concentration from a node located by a step in height ahead, into the zeroth node for the new time instant; q is the measure of transfer of the concentration from a node located by a step in height back in relation to the zeroth node, into the zeroth node for the new time instant.

The condition $p < q$ corroborates the physics of the process of air bubble liberation during propagation of oscillations.

According to the theory of Markov processes, $p + p_0 + q = 1$, which also follows from (9).

With the notation adopted, (9) assumes the form

$$w_1 = w_0 p_0 + w_2 p + w_4 q. \tag{10}$$

Equation (10) serves for the determination of the volume concentrations of air in all nodes except the lower node corresponding to the source of oscillations. At the lower node $w_1 = w_2 p + w_0(p_0 + p)$. At the top we introduce a fictitious boundary with zero concentration, corresponding to the node located beyond the free surface of the system. Physically this means that there is no transfer of air from atmosphere into the disperse system, although in reality this phenomenon can be observed to some degree.

The dimensionless column height H of the two-phase disperse system in which oscillations propagate can be determined according to the expression

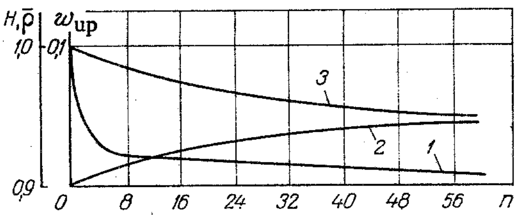

Fig. 1. Dependence of variation of the volume concentration of air bubbles at the upper node (1) of the grid corresponding to the free surface, the dimensionless mean density (2), and column height (3) on the number of periods of oscillations n. K_t = 20, M = 20; α = 0.05; A = $3.5 \cdot 10^{-3}$ m; ω = 100 sec^{-1}.

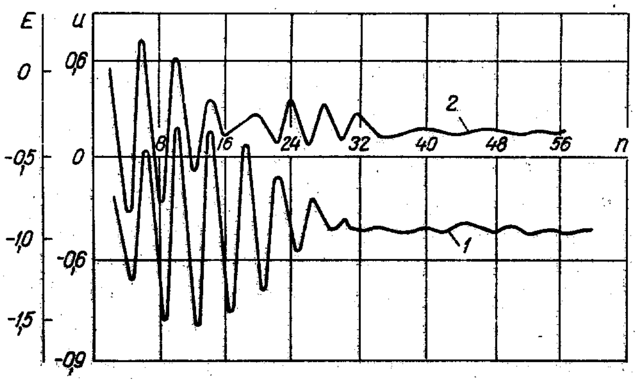

Fig. 2

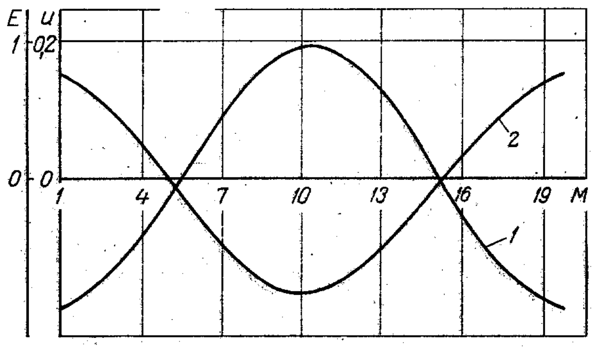

Fig. 3

Fig. 2. Dependence of the dimensionless displacement (1) and velocity (2) at the upper node of the grid corresponding to the free surface, on the number of periods of oscillation. $K_t = 20$; $M = 20$; $\alpha = 0.05$; $A = 3.5 \cdot 10^{-3}$ m; $\omega = 100$ sec^{-1}.

Fig. 3. Dependence of the dimensionless displacement (1) and velocity (2) on the number of the node of the grid along height of the column in the case of steady state oscillations for the 360° phase angle. $K_t = 20$; $M = 20$; $A = 3.5 \cdot 10^{-3}$ m; $\omega = 100$ sec^{-1}.

$$H = 1 - \frac{\sum_i q_i w_{\text{up}\cdot i}}{M} \to 1 - w_{\text{ini}}, \tag{11}$$

where w_{ini} is the initial value of the volume concentration ($w_{\text{ini}} = 0.1$); $w_{\text{up}i}$, volume concentration at the upper node corresponding to the free surface for the i-th time instant; M, number of nodes of the grid along the height of the column.

The dimensionless mean density of the two-phase system is

$$\bar{\rho} = \frac{1 - w_{\text{ini}}}{H}. \tag{12}$$

In the process of propagation of oscillations the dimesionless mean density tends to unity.

Calculations were carried out according to the algorithm obtained, and the variation of the dimensionless height of the column of the disperse system and the mean density and also the concentration of air at the nodes of the grid were determined for a variable density of the medium and velocity of sound.

In Fig. 1 we have shown the variation of concentration w of air bubbles at the upper node of the grid corresponding to the free surface, the dimensionless mean density $\bar{\rho}$, and the column height H dependent on the number n of periods of oscillations.

Analysis of the data of the figure shows that the variation of the dimensionless mean density grows continuously and tends to unity, while the dimensionless height and air bubble concentration at the upper node decrease and tend to a constant quantity.

In Fig. 2 we have represented the dependence of the dimensionless displacement E and the velocity u at the upper node of the grid corresponding to the free surface, on the number of periods of oscillation. Here derivation of values of these quantities on a computer was carried out at the end of each period of oscillation. Analysis of these results shows that the dimensionless displacement of a point of the medium corresponding to the free surface at the end of each period constitutes a fluctuation agreeing with the data presented in [11]. The pattern is symmetric about E = −1. For $K_t = 20$ the pattern of fluctuations is repeated after 40T, where T is the period of oscillation. The character of variation of the dimensionless velocity also constitutes a fluctuation which is asymmetric about zero. These fluctuations in the process of oscillation decay, with the rate of decay depending on the viscosity of the system.

In Fig. 3 we have represented the dependence of the dimensionless displacement and velocity on the number of the node along height of the column in the case of steady state oscillation for the 360° phase angle. The velocity of sound, density, and viscosity of the medium are variables.

From analysis of the data in Fig. 3 it is seen that the dimensionless displacements and velocities along the height of the column vary according to a periodic law. A comparison of these parameters in the case of other phase angles shows that characteristics of their variation correspond to the period of fluctuations.

Thus, the algorithm obtained allows us to investigate the physical relations of the process of propagation of oscillations in two-phase disperse systems.

NOTATION

v_b, relative bubble velocity; d, bubble diameter, g, acceleration due to gravity; ν, coefficient of kinematic viscosity; v, velocity of an elemental volume of the medium; x, y, coordinates; c, velocity of sound; w, volume concentration of air bubbles; D, diffusion coefficient; μ, coefficient of dynamic viscosity; ρ, density of the two-phase medium; $\rho_{l.ph}$, density of the liquid phase; $E_{l.ph}$, modulus of elasticity of the liquid phase; E_a, modulus of elasticity of air; $c_{l.ph}$, velocity of sound in the liquid phase; c_a, velocity of sound in air; ρ_a, density of air; δx, δy, step of integration along a coordinate; δt, step of integration in time; α, a coefficient characterizing the measure of friction between layers; K_t, number of time steps per period of oscillation; $\overline{\delta t}$, dimensionless step of integration in time. Indices: b, bubble; l.ph, liquid phase; a, air; up_i, the upper node for the i-th instant of time; ini, initial value.

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